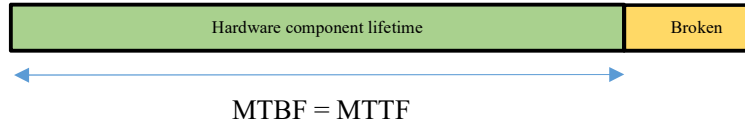


Network services availability from data sampling

1. MTBF, MTTF, MDT, MTTR, and availability

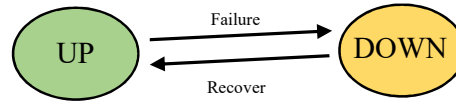
One of the most widely used metric to express something's reliability is MTBF (**Mean Time Between Failures**), though different interpretations exist on MTBF.

Traditionally, MTBF is used to express the expected lifetime of a hardware component. This assumes it's non-repairable, for instance, a lamp. In such a scenario, MTBF represents the same as the **Mean Time to Failure** (MTTF).



Typical behaviour for network services, on the other hand, is rather different. They are most often available but may become unavailable for short periods of time. In other words, they are repairable.

Under availability point of view, a service or host may be in one of two states: available or unavailable. For the sake of simplicity will call these two states, respectively, **UP** and **DOWN**.



Each time the service goes DOWN, it will be DOWN for a period of time and then will recover back to the UP state. The average duration of DOWN periods is called **Mean Down Time** (MDT), usually regarded as being the same as **Mean Time to Repair** (MTTR). One possible distinction between these two metrics is that MTTR start counting only when the actual repair job starts. Otherwise, they are pretty the same.



MTTR (or MDT) represents the average down time periods duration:

$$MTTR = \frac{\sum_{i=1}^d DownTime_i}{d} , d = \text{number of DOWN periods}$$

On the other hand, MTTF represents the average up time periods duration:

$$MTTF = \frac{\sum_{i=1}^u UpTime_i}{u} , u = \text{number of UP periods}$$

Availability expresses the relative amount of time during which the service is available (UP), thus it can be represented by:

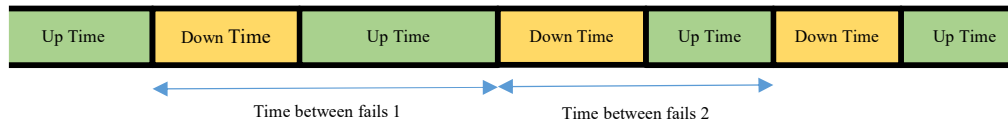
$$Availability = \frac{\sum_{i=1}^u UpTime_i}{\sum_{i=1}^u UpTime_i + \sum_{i=1}^d DownTime_i}$$

Now, only if, the number of DOWN periods is the same as the number of UP periods ($u = d$), then we can also express the service availability as:

$$Availability = \frac{MTTF}{MTTF + MTTR} ; \quad u = d$$

It's simply a matter of dividing both fraction members by the number of periods, but this requires the number of up periods (u) to be the same as down periods (d).

Analysing the service state along the time period window results in a different interpretation for MTBF. A failure is a state transition from UP to DOWN, therefore MTBF is the average time between those transitions. The time between fails comprehends a DOWN period and the following UP period (DOWN/UP cycle).



Given a sequence of DOWN/UP cycles, we can express MTBF within that period as:

$$MTBF = \frac{\sum_{i=1}^f TimeBetweenFails_i}{f} , \quad f = \text{number complete DOWN / UP cycles}$$

If we focus the analysis in a time period going from one fail start until another fail start, in other words, a sequence of complete DOWN/UP cycles, then:

$$MTBF = MTTR + MTTF ; \quad Availability = \frac{MTTF}{MTBF}$$

However, this is accurate only for the time period going from the first failure until the last failure.

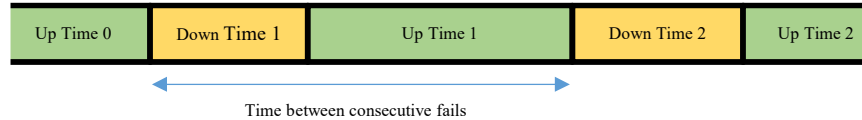
One other approach to calculate a value for MTBF in a given period is simply by dividing the total period time by the number of UP/DOWN transitions (failures).

$$MTBF = \frac{TotalTime}{f} , \quad f = \text{number of UP / DOWN transitions}$$

2. Availability analysis within a sampling time period

One first issue to be addressed when using real data is the limited time window for which data is available. Under the point of view of DOWN/UP cycles, data will lack information about for how long the service's initial state has been going on, likewise we will know the final service's state, but not for how long it will remain in that state afterwards.

The time period start and time period end cannot be regarded as state transition points, this impacts on availability metrics calculation and inevitably introduces some inaccuracy. Of course, this inaccuracy can be reduced by enlarging as far as possible the analysis time period.



On the image above respecting a sample time period, we can point three up time periods (in green), two down time periods (in orange), and four state transitions only. **Notice that from the point of view of MTBF calculation, within this time interval, we have one single useable value.**

Within this time period analysis, metrics should be calculated as follows:

Availability is the relative total UP time, **the exact value can be achieved by:**

$$Availability = \frac{UpTime0 + UpTime1 + UpTime2}{UpTime0 + UpTime1 + UpTime2 + DownTime1 + DownTime2}$$

In this scenario, an exact value for availability can't be calculated from MTTF and MTTR because the number of DOWN periods is not the same as the number of UP periods.

Mean Time to Failure (MTTF) is the average up time:

$$MTTF = \frac{UpTime0 + UpTime1 + UpTime2}{3}$$

Mean Time to Repair (MTTR) or Mean Down Time (MDT) is average down time:

$$MTTR = \frac{DownTime1 + DownTime2}{2}$$

Mean Time Between failures (MTBF) – average time between consecutive failures (UP to DOWN transitions):

$$MTBF = DownTime1 + UpTime1$$

With the provided data, this is the best MTBF value we can calculate because we only have one complete DOWN/UP cycle.

We might be then tempted to calculate availability by using this MTBF value, taking in account the single complete DOWN/UP cycle, which would be:

$$Availability = \frac{MTTF}{MTBF} = \frac{UpTime1}{DownTime1 + UpTime1}$$

Yet, this is a bad idea, it will result in a rather different value because it is using only part of the provided data (Down Time 1 and Up Time 1), thus this value is less accurate.

A more representative value for MTBF can be considered by using all provided data by dividing the total window time period by the number of failures:

$$MTBF = \frac{TotalTime}{fails} = \frac{UpTime0 + UpTime1 + UpTime2 + DownTime1 + DownTime2}{2}$$

Now availability calculated from MTBF comes as:

$$Availability = \frac{MTTF}{MTBF} = \frac{\frac{UpTime0 + UpTime1 + UpTime2}{3}}{\frac{UpTime0 + UpTime1 + UpTime2 + DownTime1 + DownTime2}{2}}$$

Nevertheless, we can see this value is two-thirds of the exact value previously calculated for availability. This error is due to the fact that in the time period there are three up time periods and only two failures. Of course, the longer is the analysis period the smaller will this error tend to be.

3. Availability analysis from constant rate periodic sampling (e.g. monitoring systems)

Most often, availability data is provided by monitoring systems. Monitoring systems perform a periodic service availability test, collecting and storing the result for each test. The **sampling rate** is, therefore, constant.

As with any constant **sampling rate**, the acquired information is limited to single points in time, each separated by a constant time period called the sampling interval or **sampling period (T)**. Depending on the sampling period, the acquired information is just a fade representation of reality. For instance, between two successful consecutive tests the service may be unavailable and failing, and that would be completely ignored.

The sampling frequency or sampling rate (f_s) is given by: $f_s = 1/T$

The approach on calculating the availability from this collected data, is by seeing each sample as representing a T duration DOWN time or UP time. So, within a time period representing all collected data, this can be calculated from the number of failed availability tests (F) and the number of success availability tests (S).

$$Availability = \frac{\sum_{i=1}^S UpTime_i}{\sum_{i=1}^S UpTime_i + \sum_{i=1}^F DownTime_i} = \frac{\sum_{i=1}^S T}{\sum_{i=1}^S T + \sum_{i=1}^F T} = \frac{S \times T}{S \times T + F \times T} = \frac{S}{S + F}$$

Regarding MTBF, MTTF, and MTTR, a more detailed analysis on data is required. This is because multiple consecutive same state tests results, must be regarded as a single same state period.

For instance, for the following sampling data:

Sampling Timing	0	T	2xT	3xT	4xT	5xT	6xT	7xT	8xT
	UP	UP	DOWN	UP	UP	UP	DOWN	DOWN	UP

It would result:

$$MTBF = 4 \times T, \text{ or in alternative, } MTBF = \frac{9 \times T}{2} = \frac{9}{2} \times T$$

In the first case the average time between two consecutive fails (only one value available in this data), in the second case by dividing the total time by the number of failures. Regarding MTTF and MTTR:

$$MTTF = \frac{2 \times T + 3 \times T + T}{3} = \frac{6}{3} \times T \quad MTTR = \frac{T + 2 \times T}{2} = \frac{3}{2} \times T$$

Availability from the original expression comes as, **this is the most accurate value**:

$$Availability = \frac{S}{S + F} = \frac{6}{9} = \frac{2}{3}$$

Depending on data, availability calculated from MTTF and MTBF is inaccurate, it would be either:

$$Availability = \frac{MTTF}{MTBF} = \frac{\frac{6}{3} \times T}{4 \times T} = \frac{1}{2}$$

, or:

$$Availability = \frac{MTTF}{MTBF} = \frac{\frac{6}{3} \times T}{\frac{9}{2} \times T} = \frac{4}{9}$$

The significant divergence results from a very limited amount of data, anyway, this is not an accurate way to calculate the availability.

The reference expression for most accurate availability calculation in these cases should always be:

$$Availability = \frac{S}{S + F}$$

4. Availability analysis from non-periodic sampling (e.g. event logging)

Another common scenario is when available data comes from event logging, this is another sampling case, but now there's no constant sampling ratio. Sampling is now randomly performed as the service is accessed, each access may be logged as successful or unsuccessful (failure).

Samples are more or less random in time, so the meaning of each sample and its relation with the reality is now even more diffuse. Nevertheless, there's no reason why a unique sample in a long period should be more significant than another unique sample in a short period.

One strategy is making all samples worth the same, this is achieved by determining the average sample rate within a period:

$$Average\ sampling\ period = \frac{Total\ sampling\ time}{Total\ number\ of\ samples}$$

Now, the previous availability calculation method can be used by taking the **average sampling period** instead of the constant **sampling period (T)**, thus, this results in the same expression as before:

$$Availability = \frac{S}{S + F}$$

Where **S** represents the total number of success samples, and **F** the total number of failure samples, both within the sampling interval.